$$Weight, W = mg$$

Where m=mass, g=gravitational acceleration

$$Density, \rho = \frac{m}{V}\left(\frac{kg}{m^3}\right)$$

Where m=mass, V =Volume

$$Specific\ volume, v = \frac{V}{m} = \frac{1}{\rho} \qquad \left(\frac{m^3}{kg}\right)$$

$$Specific\ gravity, SG = \frac{\rho}{\rho_{H_2O}}$$

Where ρ_{H2O} =1000 kg/m^3

$$P_{gage} = P_{Abs} - P_{atm} \qquad\qquad P_{vac} = P_{atm} - P_{abs}$$

Variation of pressure with depth: Apply between two points in the same fluid. Where "below" refers to point at lower elevation and "above" at higher elevation

$$P_{below} = P_{above} + \rho g |\Delta z|$$

The absolute and gage pressures in a liquid open to the atmosphere at a depth of h from the free surface are:

$$P = P_{atm} + \rho g h \qquad\qquad P_{gage} = \rho g h$$

Kinetic Energy:
Where m=mass, V=velocity
$$KE = m\frac{V^2}{2} \quad (kJ) \qquad ke = \frac{V^2}{2} \quad (\frac{kJ}{kg})$$

Potential Energy:
$$PE = mgz \quad (kJ) \qquad pe = gz \quad (\frac{kJ}{kg})$$

Where m=mass, g=gravitational acceleration, z=elevation

SPECIFIC HEAT RELATIONS FOR IDEAL GAS:

$$c_v = \left(\frac{\delta u}{\delta T}\right)_v \qquad \Delta u = u_2 - u_1 = \int_1^2 c_v(T)dT \quad (\frac{kJ}{kg})$$

$$c_p = \left(\frac{\delta h}{\delta T}\right)_p \qquad \Delta h = h_2 - h_1 = \int_1^2 c_p(T)dT \quad (\frac{kJ}{kg})$$

Variation of spec. heats with T is smooth and may be approx. as linear over small T interval. Can replace specific heat with C_{avg}, yielding:

$$u_2 - u_1 = c_{v,avg}(T_2 - T_1), \qquad h_2 - h_1 = c_{p,avg}(T_2 - T_1)$$

$$c_p = c_v + R \quad \left(\frac{kJ}{kgK}\right), \qquad k - \frac{c_p}{c_v}$$

ENERGY TRANSFER BY WORK:

Sign convention: Work done on a system = (+)
Work done by a system = (-)

Electrical Work:

When N Coulombs of electrical charge move through a potential difference V
$$W_e = VN$$

In the rate form, $\quad \dot{W}_e = VI = I^2R = \frac{V^2}{R} \qquad (W)$

Where $\dot{W}_e$ is the electrical power and I is the current.

Electrical work done during a time interval Δt:

$$W_e = \int_1^2 VI\, dt \qquad (kJ)$$

Or when V and I remain constant during interval Δt:

$$W_e = VI\,\Delta t \qquad (kJ)$$

Mechanical Forms of Work:

$$W = Fs \qquad (kJ)$$

$$W = \int_1^2 Fds \qquad (kJ)$$

Work done by a constant force, F on a body displaced a distance s

Shaft Work: $\quad \dot{W}_{sh} = 2\pi\, \dot{n}\, T \qquad (kW)$

Where $\dot{n}$ is the number of revolutions per unit time

Spring Work: $\quad W_{spring} = \frac{1}{2}k(x_2^2 - x_1^2) \qquad (kJ)$

Where x_1 and x_2 are the initial and final displacements of the spring.

Boundary Work:

$$W_b = \int_1^2 Pdv \qquad (kJ)$$

Constant P process: $W_b = P(V_2 - V_1)$

Polytropic process: $W_b = \dfrac{P_2 V_2 - P_1 V_1}{1-n}$

Polytropic Ideal Gas process: $W_b = \dfrac{mR(T_2 - T_1)}{1-n}$

Polytropic Isothermal Ideal Gas process: $W_b = PV\,ln\left(\dfrac{V_2}{V_1}\right) = mRT_o ln\left(\dfrac{V_2}{V_1}\right)$

During actual exp/comp process of gases, P and V are related by PV^n=C. Where n and C are constants therefore between 2 states, ideal gas, closed

$$C = P_1 V_1 = P_2 V_2 = mRT_0 \quad \therefore \left(\frac{V_2}{V_1}\right) = \left(\frac{P_2}{P_1}\right)$$


INTERNAL ENERGY, ENTHALPY & SPECIFIC HEATS OF SOLIDS & LIQUIDS

For an incompressible substance: $\quad c_p = c_v = c$

$$\Delta u = u_2 - u_1 = c_{avg}(T_2 - T_1) \qquad (\frac{kJ}{kg})$$

$$\Delta h = \Delta u + v\Delta P \cong c_{avg}(T_2 - T_1) + v\Delta P \qquad (\frac{kJ}{kg})$$
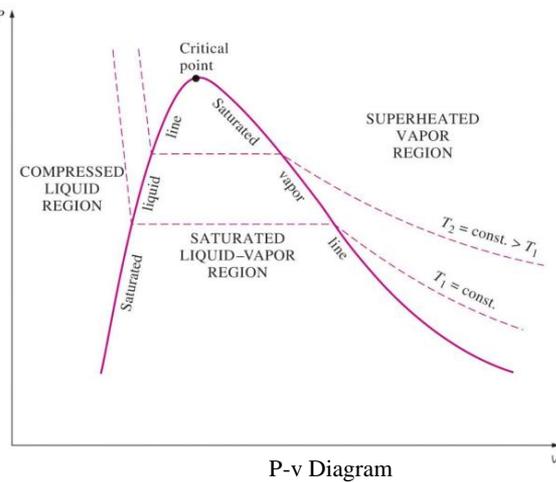
P-v Diagram
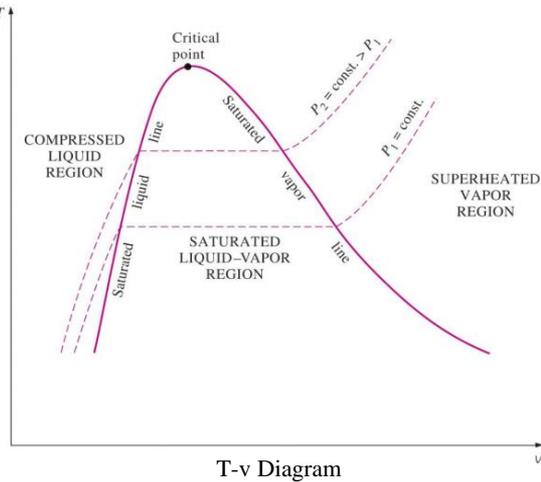
T-v Diagram

SUPERHEATED STATE

Region to the right of sat vap line & at a T above T_{cr}

To determine if S.H.

P<P_{sat} at given T $Quality, x = 1$
T> T_{sat} at given P
v>v_g at given P or T
u> u_g at given P or T
h> h_g at given P or T
s> s_g at given P or T

COMPRESSED LIQUID STATE

Region to the left of the Sat Liq line

In the absence of C.L. data – treat a C.L. as a Sat liq at given T

$$y_f \cong y_{f@T} \quad \text{Where y is v, u, s or h}$$

To determine if C.L.

P>P_{sat} at given T $Quality, x = 0$
T< T_{sat} at given P
v< v_f at given P or T
u< u_f at given P or T
h< h_f at given P or T
s< s_f at given P or T

SATURATED LIQUID-VAPOR MIXTURE STATE

Region under the dome

To determine the proportion of liquid and vapor phases in the mixture, find quality, x

$$x = \frac{m_{vapor}}{m_{total}}, \quad m_{total} = m_{liq} + m_{vap} = m_f + m_g$$

$$Quality, 0 < x < 1$$

To find v, h, u, s at state 1.

Where y is v, u, s or h

$$y_1 = y_f + x(y_g - y_f)$$

To determine if Sat Liq-vap Mixture.

$$v_f \leq v \leq v_g$$
$$u_f \leq u \leq u_g$$
$$h_f \leq h \leq h_g$$
$$s_f \leq s \leq s_g$$

IDEAL RANKINE CYCLE: The ideal cycle for vapor power cycles

1-2 Isentropic compression in a pump
2-3 Constant pressure heat addition in a boiler
3-4 Isentropic expansion in a turbine
4-1 Constant pressure heat rejection in a condenser
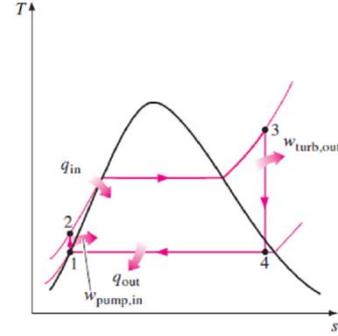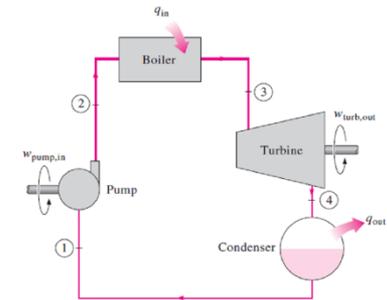
## IDEAL REHEAT RANKINE CYCLE: The ideal cycle for vapor power cycles
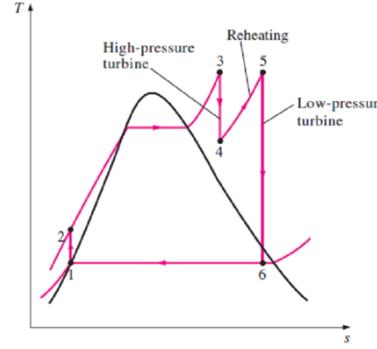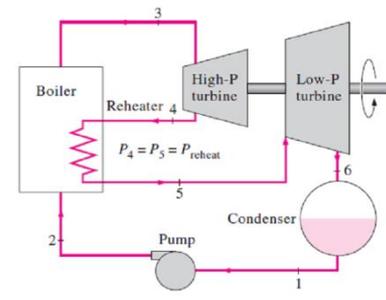
$$q_{in} = q_{primary} + q_{reheat} = (h_3 - h_2) + (h_5 - h_4)$$

$$w_{turb,out} = w_{turb,I} + w_{turb,II} = (h_3 - h_4) + (h_5 - h_6)$$

HEAT ENGINES
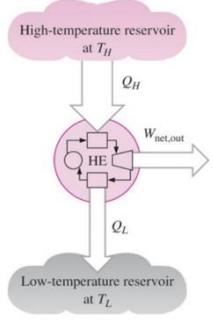
Thermal Efficiency of HE, η_{th}

$$\eta_{th} = \frac{W_{net,out}}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}}$$

$$W_{net,out} = Q_{in} - Q_{out}$$

HEAT PUMP:

The objective of a HP is to keep a warm space warm
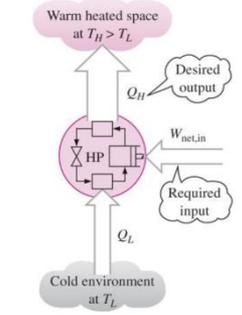
Coefficient of Performance for HP, COP_{HP}

$$COP_{HP} = \frac{Desired\ output}{Required\ input} = \frac{Q_H}{W_{net,in}}$$

$$COP_{HP} = \frac{Q_H}{Q_H - Q_L} = \frac{1}{1 - Q_L/Q_H}$$

$$COP_{HP} = COP_R + 1$$

REFRIGERATOR:

The objective of a Refrigerator is to keep a cold space cold
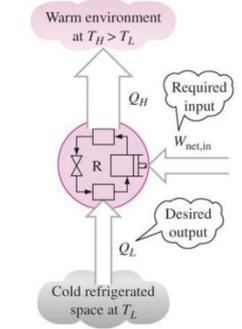
$$COP_R = \frac{Desired\ output}{Required\ input} = \frac{Q_l}{W_{net,in}}$$

$$COP_R = \frac{Q_l}{Q_H - Q_L} = \frac{1}{Q_L/Q_H - 1}$$
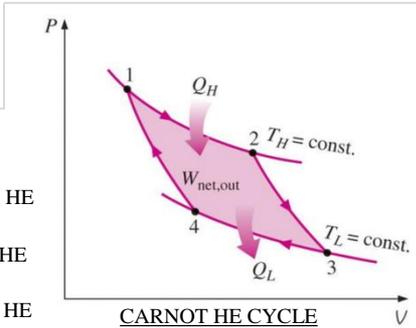
$$\eta_{th} \begin{cases} < \eta_{th,REV} & \text{Irreversible HE} \\ = \eta_{th,REV} & \text{Reversible HE} \\ > \eta_{th,REV} & \text{Impossible HE} \end{cases}$$

CARNOT HE CYCLE

ENERGY BALANCE: CLOSED SYSTEM

$$\Delta E_{sys} = E_{in} - E_{out}$$

$$\Delta U + \Delta KE + \Delta PE = E_{in} - E_{out}$$

Expanding both the left and right side of the equation:

$$m(u_2 - u_1) + \frac{1}{2}m(V_2^2 - V_1^2) + mg(z_2 - z_1) = (Q_{in} + W_{in}) - (Q_{out} + W_{out})$$

SIMPLIFY ENERGY BALANCE FOR CLOSED SYSTEM

Step 1: Define system of interest and simplify E-bal.
Step 2: If Stationary then $\Delta KE = \Delta PE = 0$
Step 3: Determine if have Q_{in} or Q_{out} if Adiabatic $\Delta Q = 0$
Step 4: Determine if have W_b, W_{paddle}, $W_{electrical}$

ENERGY BALANCE: OPEN SYSTEM, STEADY STATE

$$\Delta E_{sys} = E_{in} - E_{out} \qquad \text{Since } \Delta E_{sys} = 0 \text{ for SS} \qquad 0 = E_{in} - E_{out}$$

Expanding the equation:

$$Q_{in} + W_{in} + \sum_{in} m\left(h + \frac{V^2}{2} + gz\right) = Q_{out} + W_{out} + \sum_{in} m\left(h + \frac{V^2}{2} + gz\right)$$

MASS BALANCE: OPEN SYSTEM, STEADY STATE

$$\Delta m_{sys} = \sum_{in} m - \sum_{out} m$$

$$mass\ flow\ rate, \dot{m} = \rho V A$$

$$Volumetric\ flow\ rate, \dot{V} = VA = \dot{m}/\rho$$

Since $\Delta m_{sys} = 0$ for SS

For Steady, incompressible flow:

$$0 = \sum_{in} m - \sum_{out} m$$

$$\sum_{in} \dot{V} = \sum_{out} \dot{V}$$

ENERGY BALANCE: OPEN SYSTEM, UNSTEADY-FLOW

$$\Delta E_{sys} = E_{in} - E_{out}$$

$$\Delta U + \Delta KE + \Delta PE = E_{in} - E_{out}$$

Expanding the equation:

$$\Delta U + \Delta KE + \Delta PE = Q_{in} + W_{in} + \sum_{in} m\left(h + \frac{V^2}{2} + gz\right) - \left(Q_{out} + W_{out} + \sum_{in} m\left(h + \frac{V^2}{2} + gz\right)\right)$$

MASS BALANCE: OPEN SYSTEM, UNSTEADY-FLOW

$$\Delta m_{sys} = \sum_{in} m - \sum_{out} m \qquad \longrightarrow \qquad (m_2 - m_1) = \sum_{in} m - \sum_{out} m$$

CARNOT REFRIGERATOR

$$COP_{R,REV} = \frac{1}{T_L/T_H - 1}$$
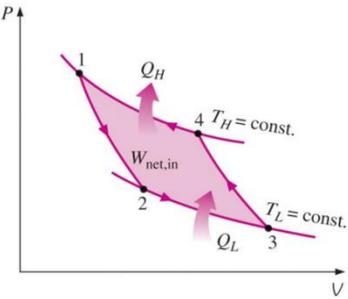
CARNOT REFRIGERATION CYCLE

CARNOT HEAT ENGINE:

$$\eta_{th,REV} = 1 - \frac{T_L}{T_H}$$

CARNOT HEAT PUMP:

$$COP_{HP,REV} = \frac{1}{1 - T_L/T_H}$$

$$\left(\frac{Q_H}{Q_L}\right)_{REV} = \frac{T_L}{T_H}$$

ENTROPY, S:

$$dS = \left(\frac{\delta Q}{T}\right)_{INT\ REV} \qquad \left(\frac{kJ}{K}\right)$$

FIND THE CHANGE IN ENTROPY:

$$\Delta S = S_2 - S_1 = \int_1^2 \left(\frac{\delta Q}{T}\right)_{INT\ REV} \qquad \left(\frac{kJ}{K}\right)$$

INTERNALLY REVERSIBLE
ISOTHERMAL HEAT TRANSFER :

$$\Delta S = \frac{Q}{T_O} \qquad \left(\frac{kJ}{K}\right)$$

INCREASE OF ENTROPY PRINCIPLE:

$$\Delta S_{SYS} = S_2 - S_1 = \int_1^2 \frac{\delta Q}{T} + S_{gen}$$

$$S_{gen} \begin{cases} > 0 & \text{Irreversible} \\ = 0 & \text{Reversible} \\ < 0 & \text{Impossible} \end{cases}$$

ISENTROPIC PROCESS:
A internally reversible, adiabatic process

$$\Delta S = 0 \quad \text{or} \quad S_2 = S_1 \quad \left(\frac{kJ}{kg\ K}\right)$$

ENTROPY CHANGE OF LIQUIDS AND SOLIDS:

$$S_2 - S_1 = \int_1^2 c(T)\frac{dT}{T} \cong c_{avg} ln\frac{T_2}{T_1} \quad \left(\frac{kJ}{kg\ K}\right)$$

Where c_{avg} is the average specific heat of the substance over the given temperature interval

SPECIAL CASE: ISENTROPIC LIQUIDS & SOLIDS

$$S_2 - S_1 = c_{avg} ln\frac{T_2}{T_1} = 0 \longrightarrow \quad T_1 = T_2$$

ISENTROPIC EFFICIENCIES OF STEADY-FLOW DEVICES

TURBINE

$$\eta_T = \frac{Actual\ Turbine\ Work}{Isentropic\ Turbine\ Work} = \frac{w_a}{w_s} \cong \frac{h_1 - h_{2a}}{h_1 - h_{2s}}$$

NOZZLE

$$\eta_N = \frac{Actual\ KE\ a\ nozzle\ exit}{Isentropic\ KE\ at\ nozzle\ exit} = \frac{V^2_{2a}}{V^2_{2S}} = \frac{h_1 - h_{2a}}{h_1 - h_{2s}}$$

COMPRESSOR

$$\eta_T = \frac{Isentropic\ Compressor\ Work}{Actual\ Compressor\ Work} = \frac{w_s}{w_a} \cong \frac{h_{2s} - h_1}{h_{2a} - h_1}$$

PUMP

$$\eta_P = \frac{w_s}{w_a} = \frac{v(P_2 - P_1)}{h_{2a} - h_1}$$

ENTROPY CHANGE OF IDEAL GAS: *CONSTANT SPECIFIC HEAT*
(Approximate Analysis: for when ΔT is small < 300°)

$$S_2 - S_1 = c_{v,avg} ln\frac{T_2}{T_1} + R \ln\frac{v_2}{v_1} \qquad \left(\frac{kJ}{kg\ K}\right)$$

$$S_2 - S_1 = c_{p,avg} ln\frac{T_2}{T_1} - R \ln\frac{P_2}{P_1} \qquad \left(\frac{kJ}{kg\ K}\right)$$

ENTROPY CHANGE OF IDEAL GAS: *VARIABLE SPECIFIC HEAT*
(Exact Analysis: for when ΔT is large & specific heats vary non-linearly w/in T range)

$$S_2 - S_1 = S°_2 - S°_1 - R \ln\frac{P_2}{P_1} \qquad \left(\frac{kJ}{kg\ K}\right)$$

ISENTROPIC PROCESS OF IDEAL GAS: *CONSTANT SPECIFIC HEAT*

$$\left(\frac{T_2}{T_1}\right)_{S=CONST} = \left(\frac{v_2}{v_1}\right)^{k-1} \qquad R/c_v = k - 1$$

$$\left(\frac{T_2}{T_1}\right)_{S=CONST} = \left(\frac{P_2}{P_1}\right)^{(k-1)/K}$$

$$\left(\frac{P_2}{P_1}\right)_{S=CONST} = \left(\frac{v_1}{v_2}\right)^{k}$$

ISENTROPIC PROCESS OF IDEAL GAS: *VARIABLE SPECIFIC HEAT*

$$\left(\frac{P_2}{P_1}\right)_{S=CONST} = \left(\frac{P_{r2}}{P_{r1}}\right)$$

$$\left(\frac{v_2}{v_1}\right)_{S=CONST} = \left(\frac{v_{r2}}{v_{r1}}\right)$$

GENERAL ENTROPY BALANCE

$$\Delta S_{sys} = \underbrace{S_{in} - S_{out}} + S_{gen}$$

Mechanisms of Entropy
Transfer = Q and m

ENTROPY BALANCE: OPEN SYSTEM

$$\Delta S_{sys} = S_2 - S_1 = \sum \frac{Q_K}{T_K} + \sum \dot{m}_i s_i - \sum \dot{m}_e s_e + S_{gen}$$

ENTROPY BALANCE: OPEN SYSTEM-STEADY FLOW

$$0 = \sum \frac{Q_K}{T_K} + \sum \dot{m}_i s_i - \sum \dot{m}_e s_e + S_{gen}$$

ENTROPY BALANCE: CLOSED SYSTEM

$$\Delta S_{sys} = S_2 - S_1 = \sum \frac{Q_K}{T_K} + S_{gen}$$

## ENTROPY BALANCE: ADIABATIC CLOSED SYSTEM

$$\Delta S_{sys} = S_{gen}$$

ENTROPY BALANCE: ADIABATIC CLOSED SYSTEM
AND SURROUNDINGS

$$S_{gen} = \sum \Delta S = \Delta S_{system} + \Delta S_{surroundings}$$

$$\Delta S_{sur} = \frac{Q_{surr}}{T_{surr}}$$

OTTO CYCLE:
Ideal cycle for spark ignition engines
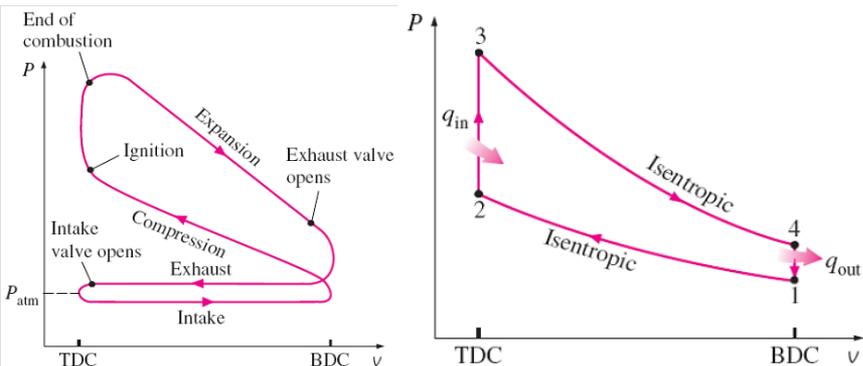
$$(q_{in} - q_{out}) + (w_{in} - w_{out}) = h_{exit} - h_{inlet}$$

$$MEP = \frac{w_{net}}{v_{max} - v_{min}}$$

$$q_{in} = u_3 - u_2 = c_v(T_3 - T_2)$$

$$q_{out} = u_4 - u_1 = c_v(T_4 - T_1)$$

$$\eta_{th,Otto} = \frac{w_{net}}{q_{in}} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{T_4 - T_1}{T_3 - T_2} = 1 - \frac{T_1(T_4/T_1 - 1)}{T_2(T_3/T_2 - 1)}$$

$$\frac{T_1}{T_2} = \left(\frac{v_2}{v_1}\right)^{k-1} = \left(\frac{v_3}{v_4}\right)^{k-1} = \frac{T_4}{T_3}$$

$$r = \frac{V_{max}}{V_{min}} = \frac{V_1}{V_2} = \frac{v_1}{v_2}$$

DIESEL CYCLE:
Ideal cycle for compression ignition engines

(b) T-s diagram

(a) P-v diagram
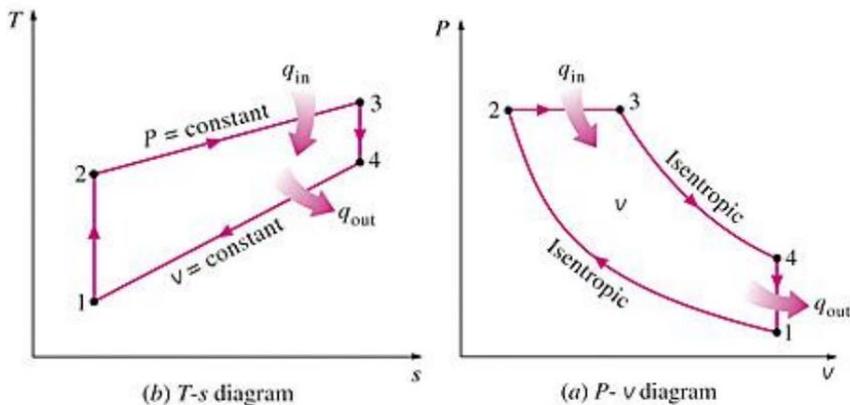
$$q_{in} - w_{b,out} = u_3 - u_2 \rightarrow q_{in} = P_2(v_3 - v_2) + (u_3 - u_2)$$

$$= h_3 - h_2 = c_p(T_3 - T_2)$$

$$-q_{out} = u_1 - u_4 \rightarrow q_{out} = u_4 - u_1 = c_v(T_4 - T_1)$$

$$\eta_{th,Diesel} = \frac{w_{net}}{q_{in}} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{T_4 - T_1}{k(T_3 - T_2)} = 1 - \frac{T_1(T_4/T_1 - 1)}{kT_2(T_3/T_2 - 1)}$$

$$\eta_{th,Diesel} = 1 - \frac{1}{r^{k-1}}\left[\frac{r_c^k - 1}{k(r_c - 1)}\right]$$

$$r_c = \frac{V_3}{V_2} = \frac{v_3}{v_2}$$

AIR STANDARD ASSUMPTIONS:
1. The working fluid is air, which continuously circulates in a closed loop and always behaves as an ideal gas.

2. All the processes that make up the cycle are internally reversible

3. The combustion process is replaced by a heat addition process from an external source.

4. The exhaust process is replaced by a heat rejection process that restores the working fluid to its initial state.

COLD AIR STANDARD ASSUMPTION:
Air has constant specific heats whose values are determined at room temp (25C)