Calculus

Write your questions and thoughts here!

3.2 Implicit Differentiation

Notes

Recall:

Explicit equation

Implicit equation

Chain Rule and Implicit Differentiation

In terms of x	In terms of y
$\frac{d}{dx}x =$	$\frac{d}{dx}y =$
$\frac{d}{dx}x^2 =$	$\frac{d}{dx}y^2 =$
$\frac{d}{dx}e^{5x} =$	$\frac{d}{dx}e^{5y} =$

Implicit Differentiation Example: Find $\frac{dy}{dx}$ for $y^2 - 5x^3 = 3y$

Step 1: Take the derivative. Each time the derivative of "y" is involved, include a $\frac{dy}{dx}$.

Step 2: Gather all terms with $\frac{dy}{dx}$ on the left side, everything else on the right.

Step 3: Factor out the $\frac{dy}{dx}$ if necessary, to create only one $\frac{dy}{dx}$ term.

Step 4. Solve for $\frac{dy}{dx}$.

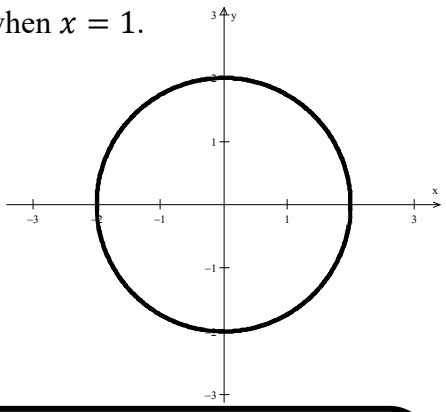
1. $y^3 - 2x = x^4 + 2y$

2. $\sin(xy) = 10x$

Write your questions and thoughts here!

Derivative at a point – implicit differentiation.

3. Find the equation of all tangent lines for $x^2 + y^2 = 4$ when $x = 1$.



Horizontal and Vertical Tangent Lines

Horizontal tangent lines exist when the slope, $\frac{dy}{dx} =$

Vertical tangent lines exist when the slope, $\frac{dy}{dx}$ is

4. Find all ***horizontal*** tangent lines of the graph $3x^2 + 2y^2 = 16$.

5. Find all ***vertical*** tangent lines of the graph $3x^2 + 2y^2 = 16$.

3.2 Implicit Differentiation

Calculus

Practice

Find $\frac{dy}{dx}$.

1. $5x^2 + 2y^3 = 4$

2. $5y^2 + 3 = x^2$

3. $\sin(x + y) = 2x$

4. $4x + 1 = \cos y^2$

5. $5x^2 - e^{4y^2} = -6$

6. $\ln(y^3) = 5x + 3$

7. $x^2 = 4y^3 + 5y^2$

8. $5x^3 - 2y = 5y^3$

9. $\ln y^2 + \cos^2 x = 1 - y$

10. $\sin\left(\frac{y}{2}\right) + e^y = 4x$

11. $x^3 + y^3 = 6xy$

12. $\frac{x}{\sin y} = 5$

13. $\ln x\, e^{3y} = 2y^2$

Find the slope of the tangent line at the given point. Show work.

14. $2 = 3x^4 + xy^4$ at $(-1, 1)$

15. $x \ln y = 4 - 2x$ at $(2, 1)$

Find the equation of the tangent line at the given point.

16. $x^2 + y^2 + 19 = 2x + 12y$ at $(4, 3)$

17. $x \sin 2y = y \cos 2x$ at $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

Find the equations of all horizontal and vertical tangent lines. Calculator allowed. Round to three decimals.

18. $x^2 + x + 2y^2 = 8$

Horizontal: _____

Vertical:_____

19. $x + y = y^2$

Horizontal: _____

Vertical:_____

20. Find the slope of the normal line to $y = x + \cos(xy)$ at $(0,1)$.

(A) 1 (B) -1 (C) 0 (D) 2 (E) Undefined

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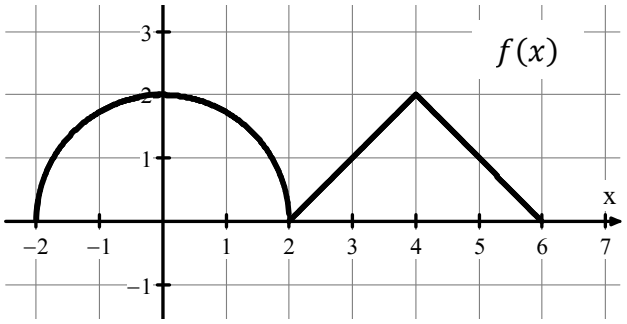
21. The graph of $f(x)$, shown below, consists of a semicircle and two-line segments. $f'(1) =$



(A) -1 (B) $-\frac{1}{\sqrt{3}}$ (C) $\frac{1}{\sqrt{3}}$ (D) 1 (E) $\sqrt{3}$

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22. Find the value(s) of $\frac{dy}{dx}$ of $x^2y + y^2 = 5$ at $y = 1$.

(A) $-\frac{3}{2}$ only (B) $-\frac{2}{3}$ only (C) $\frac{2}{3}$ only (D) $\pm\frac{2}{3}$ (E) $\pm\frac{3}{2}$